Abstracts

Origin of Vector Parasites in Numerical Maxwell Solutions

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Dispersion relations are derived for conventional finite element and finite difference approximations of four versions of the Maxwell equations in the plane: the double-curl equation; the vector Helmholtz equation; the penalty equation and the primitive, coupled Maxwell curl equations. Comparison with their analytic counterparts reveals the presence and origin of vector parasites. In each case there are no essential qualitative differences between the finite difference and finite element approaches per se; all of the issues surround the form of the differential equation underlying the discretization. For the double-curl and penalty methods, the dispersion relations are double-valued, admitting an extra, spurious dispersion surface of real. valued wavenumbers. As a result, low wavenumbers support both well-resolved and poorly resolved vector parasites. Additionally the "physical" modes in these solutions have nonzero divergence, such that satisfaction of divergence-free boundary conditions necessarily invokes the parasitic modes. The Helmholtz schemes have monotonic, single valued dispersion relations for divergence-free physical modes. Specification of divergence free boundary conditions is sufficient to guarantee the absence of parasites. The primitive schemes have single-valued but folded (nonmonotonic) dispersion relations, supporting poorly resolved vector parasites at low wavenumbers. Use of a staggered finite difference grid eliminates these parasites and results in a dispersion relation identical to that for the Helmholtz scheme. In all cases where vector parasites arise the same essential weakness in the discretized form of either the first or cross-derivative is responsible. Overall, this analysis iliuminates fatal weaknesses in the double-curl schemes considered, the reliance on a staggered mesh in the primitive schemes, and the strength of the vector Helmholtz schemes.

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